3 (Sem–5/CBCS) PHY HC 1

2023

PHYSICS

(Honours Core)

Paper : PHY–HC–5016

(Quantum Mechanics and Applications)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×7=7

   (a) Why eigenvalues and eigenfunctions of Hermite operators are very important for a quantum physicist ?

   (b) Stationary states are those for which the probability density ρ is

   (i) time-dependent

   (ii) time-independent

   (iii) space-dependent

   (iv) space-independent

Contd.

(c) Why $\psi = x^n$ is not an acceptable wave function ?

(d) Can kinetic energy and linear momentum of a quantum system be specified simultaneously ? Give reason.

(e) Write down the Schrödinger equation for a particle in a square well potential of infinite depth.

(f) The z-component of spin angular momentum can take values :

(i) $\pm h$

(ii) $\pm \frac{\hbar}{2}$

(iii) $\pm \hbar$

(iv) $\pm 2\hbar$

(g) An atomic state is denoted by ${}^4D_{5/2}$. What should be the minimum number of electrons involved for this state ?

2. Answer the following questions : $2 \times 4 = 8$

(a) Show that $[\hat{L}_x, \hat{L}_y] = i\hbar \hat{L}_z$.

(b) How much energy is released, when a $3d$ electron in the hydrogen atom makes a transition to a $2p$ state?

(c) Calculate Lande's g factor for a p-electron.

(d) Show that in a non-dispersive medium, the group velocity is equal to the phase velocity.

3. Answer ***any three*** of the following questions : 5×3=15

(a) What do you mean by dynamical variables and expectation value of a dynamical variable? Obtain quantum mechanical operators corresponding to linear momentum, kinetic energy and Hamiltonian of a system. 1+1+3=5

(b) A free particle is initially localized in the range $-a < x < a$ as :

$$\psi(x, 0) = u(x) = A, \quad -a < x < 0$$

$$u(x) = 0, \quad \text{otherwise}$$

where A, a are real and positive. Find $\psi(x, t)$.

(c) Starting from time independent Schrödinger equation in polar coordinate for hydrogen atom, show that for azimuthal angle, the probability density of electron is constant. What is its significance ?

(d) A particle in the ground state is located in one dimensional potential well of width L with absolutely impenetrable walls $0 < x < L$. Find probability of finding the particle in the region $\frac{L}{3} < x < \frac{2L}{3}$.

(e) What are identical particles ? Show that when two identical particles try to occupy same quantum state, then anti-symmetric wave function becomes zero. Why Pauli's exclusion principle is not valid for Bosons ? 1+3+1=5

4. Answer ***any three*** of the following questions : 10×3=30

(a) (i) Explain the meaning of probability current density for a quantum system. Deduce an expression for the probability current density for three dimensional motion and the law of conservation of probability density. 1+3+1=5

(ii) The wave function of a particle moving in one dimension is given to be

$$\psi(x)=\begin{cases}\sqrt{\frac{15}{a}}A\left(a^2-x^2\right) & \text{for } -a\le x\le 0\\ 0 & \text{for } |x|>a\end{cases}$$

Find the value of A that will normalise $\psi(x)$ and calculate the expectation values of x and p. 1+2+2=5

[where the notations have their usual meaning.]

(b) (i) For a linear harmonic oscillator, obtain the ground state wave function. Make a plot of the first and second energy eigenfunctions. 7+1=8

(ii) Compare the ground state classical and quantum mechanical probability of the oscillator. What happens when the quantum numbers become very large? 2

(c) From the polar equation of hydrogen atom separate the radial part and using Frobenius method find the energy states. 3+7=10

(d) (i) What is Zeeman effect? Give the explanation of normal Zeeman effect on the basis of classical theory and obtain an expression for Zeeman shift. 1+5=6

(ii) Explain why normal Zeeman effect occurs only in atoms with even number of electrons. 2

(iii) An spectrometer can resolve spectral lines separated by 0·03*nm*. How much magnetic field will have to be applied to a source of 422·7*nm* line, so that the triplet is just resolved in normal Zeeman effect? 2

(e) Differentiate between L-S and J-J coupling schemes.

The wavelengths of *D* lines of sodium are 5896*Å* and 5890*Å*. Calculate the (a) energy of the levels from which these spectral lines originate (b) separation in *eV* between the two *p*-levels in sodium atom. Given that the ionisation energy of sodium is equal to 5·13*eV*. 5+5=10

(f) Describe Stern-Gerlach experiment with a suitable diagram and explain on the basis of quantum theory.

In a Stern-Gerlach experiment silver atoms traverse a distance $0{\cdot}1m$ in a non-homogeneous field of gradient 55 Tesla m^{-1}. If the velocity of silver atom is $450ms^{-1}$, calculate the separation between the two traces on the collector plate.

[Bohr magneton = $9{\cdot}27 \times 10^{-24} JT^{-1}$, Mass of silver atom = $1{\cdot}79 \times 10^{-25} kg$]

3+5+2=10